# The 9th Romanian Master of Mathematics Competition

Day 2: Saturday, February 25, 2017, Bucharest

Language: English

**Problem 4.** In the Cartesian plane, let $\mathcal{G}_1$ and $\mathcal{G}_2$ be the graphs of the quadratic functions $f_1(x) = p_1x^2 + q_1x + r_1$ and $f_2(x) = p_2x^2 + q_2x + r_2$, where $p_1 > 0 > p_2$. The graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ cross at distinct points $A$ and $B$. The four tangents to $\mathcal{G}_1$ and $\mathcal{G}_2$ at $A$ and $B$ form a convex quadrilateral which has an inscribed circle. Prove that the graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ have the same axis of symmetry.

**Problem 5.** Fix an integer $n \geq 2$. An $n \times n$ *sieve* is an $n \times n$ array with $n$ cells removed so that exactly one cell is removed from every row and every column. A *stick* is a $1 \times k$ or $k \times 1$ array for any positive integer $k$. For any sieve $A$, let $m(A)$ be the minimal number of sticks required to partition $A$. Find all possible values of $m(A)$, as $A$ varies over all possible $n \times n$ sieves.

**Problem 6.** Let $ABCD$ be any convex quadrilateral and let $P$, $Q$, $R$, $S$ be points on the segments $AB$, $BC$, $CD$, and $DA$, respectively. It is given that the segments $PR$ and $QS$ dissect $ABCD$ into four quadrilaterals, each of which has perpendicular diagonals. Show that the points $P$, $Q$, $R$, $S$ are concyclic.

Each of the three problems is worth 7 points.
Time allowed $4\frac{1}{2}$ hours.