

NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS

Further International Selection Test, 1976

May 5th, 3½ hours

1. Through a point P in the interior of a fixed triangle ABC lines PL, PM, PN are drawn parallel to the medians through A,B,C respectively to meet BC, CA, AB at L,M,N respectively. Prove -

$$\frac{BL}{BC} + \frac{CM}{CA} + \frac{AN}{AB}$$

is constant (independent of P).

2. The real number t is a root of the equation

$$x^n + a_2 x^{n-2} + a_3 x^{n-3} + \dots + a_n = 0, \quad (n \geq 2),$$

where the coefficients are real and satisfy $-1 \leq a_r \leq 1, (2 \leq r \leq n)$.

Prove that $-\frac{1}{2}(1 + \sqrt{5}) \leq t \leq \frac{1}{2}(1 + \sqrt{5})$.

3. Prove that the equation $x^2 - 3y^2 + 5z^2 - 7t^2 = 0$

has no solutions in integers x,y,z,t other than x=y=z=t=0.

Prove that $x^2 - 3y^2 - 5z^2 + 7t^2 = 0$ has infinitely many solutions in positive integers x,y,z,t in no two of which the ratio x:y:z:t is the same.

4. Prove that it is not possible to find positive integers p and q with the property that

$$\left| \frac{p}{q} - \sqrt{7} \right| \leq \frac{2}{11q^2}$$

5. A 'figure-of-eight' curve, S, consists of two touching circles of equal radii. Show that a pair of two distinct congruent hexagons (not necessarily convex) exists with the following properties:

- (a) All the vertices of the hexagons lie on S.
- (b) Neither hexagon has all its vertices on one circle.
- (c) Neither hexagon can be obtained from the other by a single translation, a single rotation or a single reflection.